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Public Deficits and Economic Growth

Oscar Afonso, Rui Henrique Alves and Paulo B. Vasconcelos

Abstract

This paper deals with the relationship between fiscal discipline and economic growth within a monetary union. Temporary fiscal flexibility for small and less developed countries and exceptions for incentives to R&D are claimed by some authors for the case of the European Union.

A model of a monetary union between two countries to examine the impact of fiscal shocks leading to public deficits is proposed. Results suggest the pertinence of making some exceptions to the European framework, namely concerning to expenses leading to more resources devoted to R&D. In particular, this would apply to measures taken by the less developed countries, leading to an easier catching-up.

Key words: EMU, SGP, Excessive Deficits, Economic Growth, Numerical Computations.

JEL Classification: E62, H60, O33, C63.

1 Introduction

With the creation of the European Monetary Union (EMU), the framework for the definition and implementation of macroeconomic policies has dramatically changed. Member-Countries have lost their exchange rate and money-supply

*Corresponding author. Faculdade de Economia do Porto and CEMPRE – Centro de Estudos Macroeconómicos e Previsões (supported by FCT – Fundação para a Ciência e a Tecnologia, Portugal). Address: Rua Dr. Roberto Frias 4200-464 Porto, Portugal. Email: oafonso@fep.up.pt. Fax: +351 225505050. Tel: +351 225571100.

†Faculdade de Economia do Porto and CEMPRE.

‡Faculdade de Economia do Porto and CMUP – Centro de Matemática da Universidade do Porto (supported by FCT – Fundação para a Ciência e a Tecnologia, Portugal).
instruments and the use of budgetary measures has been restrained by binding
rules aimed at avoiding the creation and maintenance of excessive public deficits.

The Maastricht Treaty, 1992, established the framework for the definition
and implementation of national fiscal policies, restricting them by binding fiscal
rules, including maximum ceilings of 3% for the public deficit to GDP ratio and
of 60% for the public debt to GDP ratio, and urging for policy coordination. The
Stability and Growth Pact (SGP), 1997, reinforced the restrictive option taken
in Maastricht, introducing concrete sanctions\(^1\) and the medium-term objective
of budgetary equilibrium.

This solution has been subject of deep discussion and criticism in political
and academic circles, mainly before 1995 and after 2000. The debate should
be focused on the way in which fiscal discipline is implemented and controlled
(e.g., Buiter et al., 1993; Rubio and Figueras, 1998) rather than on the need for
fiscal discipline.

In fact, fiscal discipline is considered by a large number of authors as an es-
sential mean to avoid negative external effects arising from deficient budgetary
behaviour (e.g., De Grauwe, 2005; Baldwin and Wyplosz, 2004), namely a pos-
sible increase in the EMU interest rate, leading to possible pressures on the
European Central Bank (ECB) to implement a more expansionary monetary
policy, thus leading to an increase in inflation.

To date, the literature presents an interesting discussion on the definition
and implementation of fiscal rules, namely in the case of the EMU, positions
varying from those who support current rules (Begg et al, 2004; Buti et al,
2005) to those who claim for a significant reform (Casella, 1999; Creel, 2003;
Pisani-Ferry, 2004; Wyplosz, 2005). In particular, it shows several topics where
divergence is more profound, two of them being: (i) the possibility of consid-
ering some expenses concerning public investment (namely incentives to R&D) as
exceptions regarding the application of binding fiscal rules; and (ii) the possi-

\(^1\) The original SGP included no references to a “golden rule” in the working of the excessive
deficit procedure. However, even in that SGP version, sanctions were not truly automatic, as
the budgetary situation had to be analysed at the light of variables that helped to value the
long-term budgetary sustainability and the causes of fiscal disequilibria.
bility of differentiation of fiscal rules among countries, by considering the levels of economic divergence and the economic dimension of each country.

The debate is not yet closed, but it may already have made a relevant contribution to the recent SGP reform (European Council, 2005). The reformed SGP allows for a growing number of circumstances that lead to a non-automatic application of the sanctions, namely considering a diversified kind of public expenses that may justify the non-compliance to the “3 per cent” rule. As far as the present paper is concerned, it is relevant to note that, within that set, expenses regarding R&D are included.

In order to analyse whether or not such kind of public expenses should be treated differently and whether or not the rules for fiscal discipline should differ across countries, we develop an endogenous R&D-growth model for two countries that compose a monetary union. In each country, the production of perfectly competitive final goods uses institutions and labour together with a continuum set of quality-adjusted intermediate goods. Intermediate goods, in turn, use designs (resulting from R&D activities) under monopolistic competition. The production function, adapted from the horizontal R&D growth models developed by Acemoglu and Zilibotti (2001) and Afonso (2006), incorporates substitutability between countries in the production of final goods.

R&D can be encouraged either by a direct subsidy or through a subsidy to the production of intermediate goods, due to the close relationship between the production of intermediate goods and R&D activities. Such policies have a negative impact on the fiscal budget of each country and thus can induce adverse consequences such as those prevented by the SGP. However, these policies may reduce the technological-knowledge gap between countries. In this case, they would be crucial for an increase in the economic cohesion within the union, which could justify different fiscal rules among countries. As above mentioned, this is the main focus of the present work.

In this paper countries differ in four features. The first one relates to economic dimension, measured by labour endowments: the one with higher labour
level is called Big and the other is denominated Small. The second feature concerns domestic institutions, which are more advanced in the Big-country. The third one is related to R&D capacities, the Big-country being an innovator and the Small-country being an imitator. The last feature is an endogenous consequence of the others and relates to the domestic quality indexes measuring technological knowledge, which are higher in the Big-country.

A first approach to this problem was addressed in Afonso and Alves (2007), by considering a model where, due to complementarity between country-specific inputs in final-goods production, both countries drove only innovative R&D. As a result, this last work inhibits the consideration of feedback effects between countries, which is expected across countries that compose a monetary union.

The paper is structured as follows. Section 2 describes the model. Section 3 determines the equilibrium conditions, including the dynamic effects of a governmental intervention. Finally, section 4 offers some concluding remarks.

2 The model

2.1 Final-goods sector

Each perfectly competitive final good \( n \in [0, 1] \) is produced by one of two countries, the Small-country, \( S \), and the Big-country, \( B \). The output of \( n \), \( Y_n \), at time \( t \) is

\[
Y_n(t) = \left[ \int_0^J \left( q^{j(t)} x_{n(j,k,t)} \right)^{1-\alpha} \, dj \right] \left[ \left( (1-n)A^S_{B} L_{S,n} \right)^\alpha + \left( nA^B_{B} L_{B,n} \right)^\alpha \right],
\]

where the contribution to production of \( n \) is given by worldwide intermediate goods, the first term on the right-hand side, and by country specific labour and institutions, the second and third terms.

Each intermediate good \( j \in [0,J] \) used in the production of \( n \) is quality-adjusted; i.e., the quality upgrade is \( q > 1 \), and \( k \) is the top-quality rung at time
The quantity of $j$ with quality $k$ at $t$ can be produced by either $B$, $x_{B,n}(k,j,t)$, after a successful innovation or by $S$, $x_{S,n}(k,j,t)$, after a lower-priced successful imitation of the leading quality. Thus, the quality level of $j$ rises due to the R&D innovative activity driven by $B$. However, both countries use the top-quality intermediate goods; i.e., $k = k_B \geq k_S$, which can be produced domestically or not. In the latter case, countries import the top quality of $j$. The term $1 - \alpha$, $\alpha \in ]0, 1[$, is the aggregate intermediate-goods input share.

The term $A$ represents the level of exogenous productivity reliant on country’s institutions. As $B$’s institutions are, by hypothesis, more advanced, we consider $A_B > A_S \geq 1$, which means that an absolute productivity advantage of labour in $B$, $L_B$, over labour in $S$, $L_S$, is accounted – by assumption $L_B > L_S$.

A relative productivity advantage of either type of labour is captured by $(1 - n)$ and $n$; i.e., $L_S (L_B)$ is relatively more productive in final goods indexed by $n$ close to 0 (close to 1).

Considering the demand for each intermediate good by the producer of $n$, the production function (1) can be written as

$$Y_n(t) = \left[ \frac{p_n(t)(1 - \alpha)}{p(k, j, t)} \right]^{\frac{1-n}{n}} Q(t) \left[ (1 - n)A_S^{\frac{w}{L}} L_{S,n} + nA_B^{\frac{w}{L}} L_{B,n} \right], \quad (2)$$

where $Q(t) \equiv \int_0^t q^{k(j, t)}(1 - \alpha) dj$ is an aggregate quality index of the stock of technological knowledge and $p_n(t)$ and $p(k, j, t)$ are prices of final good $n$ and of intermediate goods $j$, respectively.

The production function merges complementarity between inputs with substitutability between countries. The optimal choice of the producer country is reflected in the equilibrium threshold $\pi$, which depends on country-specific aggregate labour and institutions,

$$\pi = \left[ 1 + \left( \frac{A_B^{\frac{w}{L}} L_B}{A_S^{\frac{w}{L}} L_S} \right)^{1/2} \right]^{-1}. \quad (3)$$

The threshold final good comes from profit maximization (by perfectly com-
petitive final goods producers and by monopolist producers of intermediate goods) and full employment equilibrium in factor markets, given the labour supply and the technological knowledge. Moreover, $\pi$ implies that the switch from one country to the other becomes advantageous. An increase in $\pi$ would mean a larger space for production in $S$, thus appearing as a measure of its relative competitiveness. For example, a larger relative supply of labour, $\frac{L_B}{L_S}$, and/or a higher relative productivity concerning the quality of institutions, $\frac{A_B}{A_S}$, results in a small $\pi$ and thus in higher number of final goods produced in $B$. Hence, optimally final goods indexed by $n \leq \pi$ ($n > \pi$) are only produced in $S$ ($B$).

The equilibrium aggregate resources devoted to intermediate-goods production, $X = X_B + X_S$, and the equilibrium aggregate output, $Y = Y_B + Y_S$, i.e., the composite final good in the union, are expressible as a function of the currently given factor levels:

$$X(t) \equiv \int_0^1 \int_0^J x_{u}(k,j,t)djdn = \left(\frac{1 - \alpha}{p(k,j,t)}\right)^{\frac{1}{\alpha}} Q(t) \sum_{i=S,B} \bar{p}_i^B(t)L_i$$

$$Y(t) \equiv \int_0^1 p_u(t)Y_u(t)dn = \left(\frac{1 - \alpha}{p(k,j,t)}\right)^{\frac{1}{\alpha}} Q(t) \sum_{i=S,B} \bar{p}_i^B(t)A_i^B L_i.$$

The aggregate or composite output of the union is obtained by integration over final goods and by normalizing its price at each time $t$ to one (numeraire). The aggregate output represents the resources of the union that are available for consumption, $C$, production of intermediate goods, $X$, and R&D, $R$: $Y = C + X + R$.

In (4) and (5), $p_B$ and $p_S$ are the index prices of final goods produced by $B$ and $S$ respectively; where $\frac{p_B}{p_S} = \left(\frac{\overline{A}}{\overline{A}}\right)^\alpha$. Equation (5) shows that: (i) economic growth is driven by the technological-knowledge progress in $B$, reflected in the aggregate quality index; (ii) the contribution of $B$ for the composite final good is higher than the contribution of $S$, since, by assumption, $L_B > L_S$ and $A_B > $
\[ A_S. \]

### 2.2 Intermediate-goods sector

Each intermediate good is produced either in \( B \) or \( S \). In the former case, it embodies the latest innovation and in the latter it arises from the imitation, at a lower cost, of the latest innovation. In either case, intermediate goods used in final-goods production embody the state-of-the-art technological-knowledge accumulated in \( B \) and summarized in \( Q \).

**Level effects in \( S \)**

The technological-knowledge gap is always favourable to \( B \); indeed

\[
Q_B(t) \equiv Q(t) > Q_S(t) \equiv \int_0^t q^{s_j(t)}(\frac{t-\bar{t}}{t}) dj. \tag{6}
\]

In a pre-union situation, the technological-knowledge level available to \( S \) is just the domestic. Thus, when compared with a pre-union situation, the improvement in the level of technological knowledge available to \( S \) – by access to the state-of-the-art intermediate goods of \( B \) – is a static gain of the union. Country \( S \) enjoys an immediate absolute and relative (to \( B \)) benefit in terms of aggregate output and factor prices since the marginal productivity of labour increases with \( Q \).\(^3\)

Moreover, the structure of final-goods production is also affected, but in both countries. In fact, in pre-union each country produces all final goods, while after the union there is a final-goods specialisation determined by differences in domestic labour endowments and domestic quality of institutions – see (3).

\(^2\)Since \( S \) is not too backward (i.e., an appropriate taxonomy for our \( B \) and \( S \) countries would be developed versus developing, rather than developed versus underdeveloped), it is predictable that inter-country differences in prices of final goods are of second order. Moreover, in the context of a monetary union, with single currency and common market, prices of tradable goods tend to be very similar, as well as national inflation rates.

\(^3\)Note that, even under the union, not all innovations have been imitated yet at each \( t \).
Limit pricing of intermediate goods

Since the production of intermediate goods and R&D are financed by the resources saved after consumption of the composite final good, the simplest hypothesis is to consider that, in each country, the production function of intermediate goods is identical to the composite final good specified by equations (1) and (5). Given this convenient simplification, the marginal cost of producing each $j$ equals the marginal cost of producing the aggregate output, which, due to perfect competition in the final-goods sector, equals the price of the aggregate output; i.e., 1 (numeraire). Thus, the marginal cost of producing $j$ is independent of its quality level and is identical across all domestic industries. Therefore, the after subsidy marginal cost of intermediate goods production in $S$ is $1 - z_x$.5

The production of an intermediate good involves a start-up cost of R&D, either in a new design invented in $B$ or in its imitation (by reverse engineering) in $S$. This investment can only be recovered if profits are positive within a certain period in the future. This is assured by costly R&D together with domestically enforced patents, which protect, inside but not outside the country, the leader firm’s monopoly of that quality good, while at the same time disseminating obtained knowledge to other domestic firms. Thus, knowledge of how to produce the latest quality good is public (non-rival and non-excludable) inside and semi-public (non-rival and partially non-excludable) outside each country.

Even without inter-country protection of patents, the current producer of $j$ enjoys some inter-country monopoly power: for example, if the producer is from $B$, thus being challenged by either another producer in $B$ or by an imitator is $S$, monopoly is temporarily assured by enforced patents in $B$ and by costly imitation in $S$. However, the length and magnitude (measured by the mark-up) of the monopoly power are shortened by the union – in pre-union situation the

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4 Or, equivalently, that the composite final good is the input in the production of each intermediate good (e.g., Barro and Sala-i-Martin, 1997).

5 Alternatively, we could consider that any of the governments can subsidise intermediate goods production, by means of an ad-valorem subsidy $z_x$, which can be country-specific; i.e., $z_{x,S}$ in $S$ and $z_{x,B}$ in $B$ and $z_{x,S} > z_{x,B}$. 

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8
<table>
<thead>
<tr>
<th>$t - dt$</th>
<th>$t$</th>
<th>Share in $j$’s production at $t$</th>
<th>$p(j)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$ exports quality $k$</td>
<td>$B$ innovates and exports quality $k + 1$</td>
<td>$\Xi(1 - \gamma)$</td>
<td>$q$</td>
</tr>
<tr>
<td>$B$ exports quality $k$</td>
<td>$S$ imitates and exports quality $k$</td>
<td>$1 - \Xi$</td>
<td>$1$</td>
</tr>
<tr>
<td>$S$ exports quality $k$</td>
<td>$B$ innovates and exports quality $k + 1$</td>
<td>$\Xi\gamma$</td>
<td>$q(1 - z_x)$</td>
</tr>
</tbody>
</table>

Table 1: Limit pricing of each intermediate good

producer in $B$ can be only challenged by other producer in $B$ and not by an imitator in $S$ with lower effective marginal cost (due to the subsidy).

Following Grossman and Helpman (1991, ch. 12), we consider that limit pricing by each leading monopolist is optimal. In general, depending on whether $q(1-\alpha)$ is greater or lesser than 1, the leader of each $j$ would, respectively, use the monopoly pricing, $\frac{1}{1-\alpha}$, or the limit pricing, $q$, to capture the entire domestic market (e.g., Barro and Sala-i-Martin, 2004, ch. 7). To rule out monopoly pricing, we assume that the size of each quality, $q$, is not large enough.

Under the union case three possible sequences of successful R&D outcomes and their limit pricing consequences at $t$, given quality $k$ at $t - dt$ (Table 1).

The first mark-up is the highest – the entrant in $B$ competes with an incumbent in $B$ at the same (effective) marginal cost but with better quality. The second one is smaller – the entrant in $S$, with lower effective marginal cost, competes in the same quality rung with an incumbent in $B$. Compared with the first, the third mark-up is again smaller, but due to a different reason – the entrant in $B$ improves quality as in the first case, but competes with an incumbent with lower effective marginal cost.

Now we define a price index for intermediate goods, at each $t$, as a weighted average of the limit prices:

$$\bar{p}(j) = 1 + \Xi(q - 1) - \Xi\gamma q z_x,$$  \hspace{1cm} (7)
where: (i) \( \Xi \) represents the share of intermediate goods produced in \( B \) (and thus \( 1 - \Xi \) in \( S \)); (ii) \( \gamma \) is the share of intermediate goods produced in \( B \) having overcome imitator competition (\( 1 - \gamma \) is the share of intermediate goods produced in \( B \) having overcome innovator competition);

6 and (iii) \( z_x \) is a governamental subsidy towards intermediate-goods production in \( S \). This subsidy, paid as a fraction ad-valorem to each firm, is required in order to allow for the entry of \( S \)'s intermediate goods in the union market; i.e., to allow producers in \( S \) of the same quality rung \( k \) to under-price its competitor in \( B \).

2.3 R&D sector

As suggested by (5) R&D drives economic growth in \( B \) and in \( S \).

Following Afonso and Alves (2007), let \( I_B(k, j, t) \) denote the instantaneous probability at \( t \) – a Poisson arrival rate – of successful innovation in the next higher quality \( k(j, t) + 1 \) in \( j \),

\[
I_B(k, j, t) = y_B(k, j, t) \beta_B q^{k(j, t)} \zeta_B^{-1} q^{k(j, t)(-\frac{1}{2})} \frac{1}{L_B + L_S} \tag{8}
\]

where:

(i) \( y_B(k, j, t) \) is the flow of final-good resources devoted to R&D in \( j \) in \( B \);

(ii) \( \beta_B q^{k(j, t)}, \beta_B > 0 \), is the positive learning effect of accumulated public knowledge from past successful R&D – e.g., Grossman and Helpman (1991, ch. 12) and Connolly (2003);

(iii) \( \zeta_B^{-1} q^{k(j, t)(-\frac{1}{2})}, \zeta_B > 0 \), is the adverse effect caused by the increasing complexity of quality improvements in \( j \) (e.g., Kortum, 1997, and Dinopoulos and Segerstrom, 2007);

(iv) \( \frac{1}{L_B + L_S} \) is the adverse effect of market size, capturing the idea that the difficulty of introducing new quality-adjusted intermediate goods and replacing

\[\text{The specification of these shares as functions of the probabilities of successful R&D follows Dinopoulos and Segerstrom (2007), such that the share of intermediate goods produced in } B \text{ increases with the probability of successful innovation and decreases with the probability of successful imitation.}\]

\[\text{Since } B \text{ is more developed, it can be alternatively considered that } \beta_{BS} > \beta_{SB}; \text{ i.e., that } B \text{ has a better innovation capacity than } S.\]
old ones is proportional to the size of the market due to coordination among agents, organisational and transportation costs, processing of ideas, information and marketing (e.g., Alesina and Spolaore, 1997, Dinopoulou and Segerstrom, 1999, and Dinopoulou and Thompson, 1999).

The positive learning effect, (ii), is thus modelled in such a way that, together with the adverse effect, (iii), it totally offsets the positive influence of the quality rung on the profits of each intermediate good leader firm, as we can see below.

Thus, R&D activities in $B$ result in innovative designs for the production of intermediate goods, which increase their quality. The designs are domestically patented and the leader in each $j$, which produces according to the latest patent, uses limit pricing to assure monopoly. The value of the leading-edge patent relies on the profit-yields accruing during each $t$ to the monopolist, and on the duration of the monopoly power. The duration, in turn, depends (i) on the probability of a new innovation, which creatively destroys the current leading-edge design in the lines of the Schumpeterian models (e.g., Aghion and Howitt, 1992); or (ii) on the probability of an imitation in $S$ (e.g., Grossman and Helpman, 1991, ch. 12).

Following Afonso and Vasconcelos (2007), the probability of successful imitation, $I_S(k, j, t)$, of the current higher quality $k(j, t)$ in $j$ is also required:

$$I_S(k, j, t) = y_S(k, j, t) \beta_S q^{k(j, t)} S_S^{-1} q^{k(j, t)(-\frac{1}{\sigma})} \cdot \frac{1}{L_B + L_S} \left[ \tilde{Q}(t) \left( 1 - \tilde{Q}(t) \right) \right]^{-\sigma + \tilde{Q}(t)} \tag{9}$$

where:

(i) $y_S(k, j, t)$ is the flow of final-good resources devoted to R&D in $j$ in $S$

(ii) $\beta_S q^{k(j, t)}$, $0 < \beta_S < \beta_B$, $k_S < k$; i.e., we consider the learning effect of accumulated imitations lower than the learning effect of accumulated innovations;

(iii) $S_S^{-1} q^{k(j, t)(-\frac{1}{\sigma})}$, $0 < \sigma_S < \sigma_B$; i.e., we consider that the adverse effect
caused by the increasing complexity of quality improvements in $j$ is lower in the imitation situation;

(iv) $\frac{1}{\sigma + \tilde{Q}(t)}$ is the adverse effect of market size, as explained above;

(v) $\tilde{Q}(t) \left(1 - \tilde{Q}(t)\right)^{-\sigma + \tilde{Q}(t)}$ is a catching-up term, specific to $S$, which includes positive effects of technological-knowledge backwardness, since $\sigma > 0$ and $0 < \tilde{Q}(t) = \frac{Q_S(t)}{Q(t)} < 1$ is the relative technological-knowledge level of $S$. The catching-up term is quadratic over the range of interest, and, once affected by the exponent function $-\sigma + \tilde{Q}(t)$, yields an increasing (in the technological-knowledge gap) advantage of backwardness — where the size of $\sigma$ affects how quickly the probability of successful imitation decreases as the technological-knowledge gap falls.

In the absence of the union, the R&D process in $S$ mimics the R&D process in $B$, but less efficiently, i.e., with $k_S \leq k$ in (8). Since $S$ is less developed, but not too backward, we assume that there are some intermediate goods, but not all, for which $k_S < k$, implying that even in the absence of trade there are some state-of-the-art intermediate goods produced in both countries (i.e., for which $k_S = k$). Once $S$ has access to all the best quality intermediate goods due to the union, it becomes an imitator, improving the probability of successful R&D. Thus, R&D activities in $S$, when successful, result in imitation of current worldwide best qualities.

In the presence of the union, in addition to the direct effect on the capacity of imitation, the level effect increases the aggregate income in $S$ and thus more resources are available for R&D. Moreover, we will allow any of the governments to subsidise R&D activities directly, by means of an ad-valorem subsidy $z_r$, which can be country-specific (i.e., $z_{r,S}$ in $S$ and $z_{r,B}$ in $B$).

### 2.4 Consumers

A time-invariant number of heterogeneous individuals in the union — continuously indexed by $\alpha \in [0,1]$ — decide the allocation of income, which is partly spent on consumption of the composite final good, and partly lent in return for fu-
ture interest. For simplicity, we consider an exogenous threshold individual \( \bar{a} \), smaller than 0.5, since \( L_B = \int_{\bar{a}}^{1} da > L_S = \int_{0}^{\bar{a}} da \): individuals \( a > \bar{a} \) are located in \( B \), whereas individuals \( a \leq \bar{a} \) are located in \( S \).

Independently of the individual \( a \), the path of the consumption is given by the standard Euler equation

\[
\tilde{c}(a, t) = \tilde{c}(t) = \hat{C}(t) = \frac{(1 - \tau K) r(t) - \rho}{\theta}
\]  

(10)

where:

(i) \( \tilde{c}(a, t) \) and \( \hat{C}(t) \) are, respectively, the growth rate of \( c(a, t) \), which is the amount of consumption of the composite final good by the individual \( a \) at \( t \), and the growth rate of \( C(t) = \int_{0}^{1} c(a, t)da \), which is the amount of aggregate consumption at \( t \);

(ii) \( r \) and \( \tau K \) are, respectively, the interest rate and the ad-valorem tax on assets, and they are equal in both countries;\(^8\) for \( r \) this is a natural consequence of the monetary union and for \( \tau K \) is a result of perfect mobility of capital within the union.\(^9\)

(iii) \( \rho > 0 \) is the homogeneous subjective discount rate;

(iv) \( \theta > 0 \) is the inverse of the inter-temporal elasticity of substitution.

The Euler equation (10) arises from maximising the infinite horizon lifetime utility of the individual \( a \) subject to the respective budget constraint. The objective function is given by the integral of a discounted constant elasticity of substitution (CIES) utility function,

\[
\int_{0}^{\infty} \frac{c(a, t)^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt,
\]

(11)

and the budget constraint, expressed as savings + consumptions = income, at each \( t \) is

\(^8\)Also note that due to arbitrage in the domestic assets market, \( r \) depends on \( t \), but is independent from the individual.

\(^9\)Together with ad-valorem tax on wages \( \tau_w \), \( \tau_K \) may be used by the government for fiscal policies purposes (in particular, as a means of financing, at least partially, the costs of the above mentioned subsidies).
\[
\dot{K}(a,t) + c(a,t) = (1 - \tau_K) r(t) K(a,t) + \begin{cases} 
(1 - \tau_{w,S}) w_S(a,t), & \text{if } a \leq \pi \\
(1 - \tau_{w,B}) w_B(a,t), & \text{if } a > \pi
\end{cases}.
\] (12)

Savings, \( \dot{K} \), consist of accumulation of financial assets \(- K \), with return \( r \) in the form of public debt owned by individuals and in the form of ownership of the firms that produce intermediate goods in monopolistic competition.\(^{10}\)

The wage of individual \( a \) at time \( t \), \( w \), is \( w_S \) or \( w_B \) according to the country. Parameters \( \tau_{w,S} \) and \( \tau_{w,B} \) are the country-specific ad-valorem taxes on wages, which, together with \( \tau_K \), may be used by the government for fiscal policies purposes (in particular, as a means of financing, at least partially, the costs of the above mentioned subsidies).

### 2.5 Government

Since, in addition, to firms and individuals, both economies can also be influenced by domestic government policies, in order to finalise the characterisation of both economies, a description of the government’s budget is now in order.

As already stated, the government of each country may intervene by imposing taxes on wages and/or on financial assets and by subsidising the production of intermediate goods and/or R&D activities. If necessary, the government may run a public deficit by issuing public debt sold to individuals.

The budget surplus, \( BuS \), of \( S \) and \( B \) is given respectively by:

\[
(1 + r(t)) BuS_i(t) = \tau_K r(t) \int_{\inf}^{\sup} K(a,t) da + \\
+ \tau_{w,i} \int_{\inf}^{\sup} w_i(a,t) da - z_{x,i} X_i(t) - z_{r,i} R_i(t) - r(t) D_i(t);
\] (13)

where \( \inf = 0 \) and \( \sup = \bar{a} \) for \( i = S \), \( \inf = \bar{a} \) and \( \sup = 1 \) for \( i = B \). \( D_S \) and \( D_B \) represent the public debt in \( S \) and \( B \), respectively. Thus, the first and

\(^{10}\)The value of these firms, in turn, corresponds to the value of patents in use.
second terms on the right-hand side represent government tax revenue from 
assets income and from labour income, respectively, while the third and fourth 
terms represent government expenditure on subsidies for intermediate goods 
and for R&D, respectively, and the last term relates to interest paid on public 
debt.\footnote{We know that \( BuS(t) = \text{Taxes}(t) - \text{Subsidies}(t) - r(t)D(t - dt) \) and \( D(t) = D(t - dt) - BuS(t) \), which gives \( D(t - dt) = D(t) + BuS(t) \), leading to equations (13). For a matter of simplification and in order to focus the discussion just on direct subsidies to R&D, we will assume \( z_{r,B} = 0 \) and no taxes.}

We will be particularly interested in the effects of higher levels of subsidies to 
R&D in \( S \), regarding an eventual convergence towards the level of development 
of \( B \). Such an effect would become an argument in favour of different fiscal 
rules among countries in the union, namely a temporary authorisation for higher 
ratios between public deficit and GDP in the less developed countries. In the 
same context, this would also be an argument in favour to certain pertinent 
factors included in the SGP reform.

\section{Equilibrium}

We proceed by analysing the equilibrium dynamics of technological knowledge, 
which drives economic growth. The interaction effects between \( B \) and \( S \), arising 
from the union, play a crucial role in the dynamic general equilibrium.

\subsection{Equilibrium R&D}

Given the functional forms (8) and (9) of the probabilities of success in R&D, 
free-entry equilibrium is defined by the equality between expected revenue and 
resources spent. For example, taking the case of imitation by \( S \) into account, 
such equality takes the form

\begin{equation}
I_S(k, j, t)V_S(k, j, t) = (1 - z_{r,S})y_S(k, j, t),
\end{equation}

where
\[ V_S(k,j,t) = \frac{\Pi_S(k,j,t)}{r(t) + I_D(k,j,t)} \]

is the expected current value of the flow of profits to the monopolist producer of \( j \).\(^{12}\) The expected flow of profits, \( \Pi_S \), depends on the amount at each \( t \), on the interest rate, and on the expected duration of the flow, which is the expected duration of technological-knowledge leadership. Such duration, in turn, depends on the probability of a successful innovation in \( B \), which is the potential challenger.

Bearing in mind the second sequence in Table 1, the amount of profits at \( t \), for the monopolist producer of \( j \), using an imitation of quality \( k \),

\[
\Pi_S(k,j,t) = \left( \frac{1 - \alpha}{1 - z_{x,S}} \right)^{\frac{1}{\xi}} q^{k(j,\beta)} \left( \frac{1 - \alpha}{1 - z_{r,S}} \right)^{\frac{1}{1 - \xi}} \left[ L_S (A_{SPS}(t))^{\frac{n}{\xi}} + L_B (A_{BPB}(t))^{\frac{n}{1 - \xi}} \right],
\]

depends on the marginal cost, on the mark-up, and on the world demand for intermediate good \( j \) by the final-goods producers.

Plugging (16) into (15) and then (15) and (9) into (14) and solving for \( I_B \), the equilibrium probability of a successful innovation in \( B \) – given the interest rate and the price indexes of final goods – is

\[
I_B(t) = \frac{\beta_S}{S_S} \left[ \tilde{Q}(t) \left( 1 - \tilde{Q}(t) \right) \right]^{-\sigma + \tilde{Q}(t)} \left( \frac{1 - z_{x,S}}{1 - z_{r,S}} \right)^{\frac{1}{\xi}} \left( 1 - \alpha \right)^{\frac{1}{1 - \xi}}.
\]

The equilibrium \( I_B \) turns out to be independent of \( j \) and \( k \), due to the removal of scale of technological-knowledge effects.\(^{13}\)

\(^{12}\)In other words, \( V_S \) is the value of the monopolist firm owned by consumers in \( S \) or the market value of the patent.

\(^{13}\)The positive influence of the quality rung on profits and on the learning effect is exactly offset by its influence on the adverse effect induced by increasing complexity – see the
Since the probability of successful innovation, as a Poisson arrival rate, determines the state-of-the-art technological-knowledge progress, equilibrium can be translated into the path of technological knowledge in B, which allows S to benefit as well. The relationship turns out to yield the following expression – where (17) is plugged in – for the equilibrium rate of growth of technological knowledge:

\[ \dot{Q}(t) = \left\{ \frac{\beta_S}{\xi_S} \left[ \tilde{Q}(t) \left( 1 - \tilde{Q}(t) \right) \right]^{\frac{k}{1-k}} \tilde{Q}(t) \left( \frac{1 - z_{s,S}}{1 - z_{r,S}} \right)^{\frac{k}{1-k}} (1 - \alpha)^{\frac{k}{1-k}} \right\} \cdot \frac{\tilde{Q}(t)(1 - \alpha)^{\frac{k}{1-k}}}{L_S + L_B} \left[ L_S (A_{S\rho S}(t))^{\frac{1}{1-k}} + L_B (A_{B\rho B}(t))^{\frac{1}{1-k}} \right] - r(t) \left\{ q \left( \frac{1 - \alpha}{1} \right) - 1 \right\}. \]  

(18)

It is clear in (18) that there are feedback effects from imitation to innovation. That is, the positive level effect from B to S – the access to the state-of-the-art intermediate goods increases production and thus resources available to imitation – feeds back into the innovator, affecting its technological knowledge through creative destruction. Since subsidies in S improve technological knowledge in B, they improve not only the domestic level of development but the level of development of the union.

The equilibrium aggregate resources devoted to R&D, \( R(t) = R_S(t) + R_B(t) \), where \( R_S(t) = \int_0^1 y_S(k, j, t) dj \) and \( R_B(t) = \int_0^1 y_B(k, j, t) dj \), is

\[ R(t) = \left\{ \frac{\xi_S}{\beta_S} \left[ \tilde{Q}(t) \left( 1 - \tilde{Q}(t) \right) \right]^{\sigma - \tilde{Q}(t)} I_S(t) + \frac{\xi_B}{\beta_B} I_B(t) \right \} (L_S + L_B) Q(t). \]  

(19)

Like for \( Y \) and \( X \) – see (4) and (5) –, equation (19) shows that also resources devoted to R&D are positively related with the technological knowledge in B.

exponents of \( q \) in (16) and in (9)-(ii) and (iii).
3.2 Steady state

In each country and thus in the union, the aggregate final good, $Y$, is used for consumption, $C$, and savings, which in turn are allocated between production of intermediate goods, $X$, and R&D, $R$. Since both countries have the access to the state-of-the-art intermediate goods and they have the same technology of production of final goods – except for the levels of exogenous productivity and labour endowments –, in steady state they have differences in the levels but not in the growth rates. The common and stable steady-state growth rate is thus equal to growth rate of the technological knowledge in $B$, because $Y$, $X$, $R$ and $C$ are all constant multiples of $Q$. Through the Euler equation (10), the steady state interest rates, $r^* (= r^*_B = r^*_S)$, are also equalized between countries.

The common and stable steady state growth rate, designed by $g^* (= g^*_B = g^*_S)$ is thus:

$$g^* = \check{Q}^* = \check{Y}^* = \check{X}^* = \check{R}^* = \check{C}^* = \check{\varepsilon}^* = \frac{(1 - \tau_K) r^* - \rho}{\theta},$$

(20)

implying constant steady-state levels of $\check{Q}$. Indeed, while entire convergence in available technological knowledge is instantaneous with the union (level effect), domestic levels may not converge completely; that is, $\check{Q}$ may stay below one.

Clearly, R&D drives steady-state endogenous growth. The intensity of the driving force is, in turn, influenced by the union. In order to look at the steady-state effects of the union we must investigate $g^*$ further. To this end, since $g^*$ results directly from plugging $r^*$ into (10), it is sufficient to compare the steady-state interest rate:

$$r^* = \left[ (1 - \tau_K) + \theta \left( g^{(1-a)} - 1 \right) \right]^{-1} \left\{ \theta \frac{\beta_S}{\gamma_S} \left[ \check{Q}^* \left( 1 - \check{Q}^* \right) \right]^{\sigma + \check{Q}^*} \frac{\check{Q}(t)}{L_S + L_B} \right\}$$

$$\times \left( \frac{(1 - z_{x,s})^{\frac{1}{\sigma}}}{1 - z_{r,s}} \right) (1 - a)^{\frac{1}{\sigma}} \left[ L_S \left( A_{SP S}^* \right)^{\frac{1}{\sigma}} + L_B \left( A_{BP B}^* \right)^{\frac{1}{\sigma}} \right] \left( g^{(1-a)} - 1 \right) + \rho \right\},$$

(21)
obtained by setting the growth rate of consumption in (10) equal to the technological-knowledge growth rate in $B$ given by (18), with the one that would prevail in a pre-union steady state. Bearing in mind that the advantages of backwardness term vanish from the probability of successful imitation (9) in the absence of the union,\footnote{In pre-union the R&D process in $S$ mimics the R&D process in $B$, but less efficiently.} the increment in the steady-state interest rate, from pre-union to union, depends on the difference

$$\left[\tilde{Q}^* \left(1 - \tilde{Q}^* \right)\right]^{-\sigma + \gamma \frac{1}{q} \tilde{Q}^* \left(1 - z_{x,S} \right)^{\frac{1}{1-\alpha}}} \cdot \left[ L_S (A_{S}\pi_S)^{\frac{1}{2}} + L_B (A_{R}\pi_B)^{\frac{1}{2}} \right] - (1 - z_{x,S})^{\frac{1}{1-q}} \left( q - 1 \right) L_S A_S^{\frac{1}{q}}.
$$

While evaluation of (22) requires solving for transitional dynamics through calibration and simulation, we can, however, emphasize five ways, in addition to the level effects, through which the union influences, in opposite directions, steady-state growth.

The first way in which the union influences steady-state growth is the positive catching-up effect on the probability of successful imitation. The advantages of backwardness are only obtained in the presence of the union (or alternatively under trade). Through the feedback effect described above, the probability of successful innovation is also affected and thus the steady-state growth rate – see (17) and (18).

The second way is the positive spillovers from $B$ to $S$. Each innovation in $B$ tends to lower the cost of imitation by $S$ because the backwardness advantage is strengthened with each improvement of the technological-knowledge frontier.

The third way is the positive effect arising from market enlargement, which encourages R&D activities by effecting the respective profitability, as (16) shows.

The fourth – counteracting – channel is the monopolistic competition mark-up. The monopolist in $B$ loses profits with the entry into the union: the average mark-up between the first and third situations in Table 1 is smaller than $q$, which
is the pre-union mark-up. The reason for this is that in pre-union successful innovators are protected from international competition. Once engaged in the union and imitation becomes profitable, profit margins in $B$ are reduced, which discourages R&D activities.

The fifth — counteracting as well — way through which the union affects steady-state growth, is that firms in $S$ have to support the R&D cost of state-of-the-art intermediate goods, possibly several quality rungs above (and thus more complex) their own experience level in pre-union. This is reflected in the presence of $\bar{Q}^*$ in (22).

The effect of the union on the steady-state growth rate is, thus, ambiguous. However, the comparative statics of the steady-state interest rate $r^*$ in (21) — or alternatively of $g^*$ — is not affected by such ambiguity because the reported changes refer to steady-state under union. $r^*$ and $g^*$ are affected by the levels of exogenous variables and parameters, which is expected in an endogenous growth model. In particular, both countries’ exogenous levels of productivity ($A_B$ and $A_S$) and parameters of R&D technology ($\beta_S$ and $\zeta^{-1}_S$) improve the common growth rate through their positive effect on the profitability of R&D, as (16) and (17) demonstrate.

The impact on steady-state growth of an increase in the subsidy towards the production of intermediate goods in $S$, $z_{e,S}$, results from the combination of typical Schumpeterian-R&D effects: it implies a smaller effective marginal cost of production for the intermediate-goods producers in $S$, thereby encouraging imitative R&D and innovative R&D (feedback effect). The impact of a subsidy to R&D in $S$, $z_{r,S}$, is more direct: it decreases the cost of imitation, thus encouraging it, also as innovative R&D though feedback effects.

### 3.3 Transitional dynamics induced by government intervention

In this section, we solve numerically the transitional dynamics towards the steady state in order to illustrate the effects of a governmental intervention
on the country-specific technological knowledge. Using such results, we analyse whether (or not) different fiscal rules may be needed in order to offset divergences in development among member-countries within a monetary union and whether (or not) in such context incentives to R&D should be excluded from the definition of an "excessive" public deficit.

By considering utility maximization by individuals and the path of technological-knowledge in each country, we can solve the model by numerical integration, using the explicit Runge-Kutta Dormand-Prince pair. This way, the transitional dynamics is displayed, bearing in mind the initial condition \( \tilde{Q}(0) = 0.60 \) and the following set of baseline parameter values and labour endowments:

\[
A_B = 1.20, \ A_S = 1.00, \ \alpha = 0.60, \ q = \frac{1}{1+\alpha}, \ \beta_B = 1.60, \ \beta_S = 1.20, \ \varsigma_B = 5.00, \ \varsigma_S = 4.00, \ \sigma = 0.85, \ z_{x,B} = z_{x,S} = z_{r,B} = z_{r,S} = \tau_{w,B} = \tau_{w,S} = \tau_K = 0.00, \ \theta = 1.05, \ \rho = 0.015, \ MC_B = 1, \ MC_S = 0.80, \ H_B = 1.53, \ L_B = 1.53, \ H_S = 1.00, \ L_S = 1.00. \tag{15} \]

We consider four scenarios: (i) Scenario 1 (Sc1) – no governmental intervention; (ii) Scenario 2 (Sc2) – an ad-valorem subsidy to the production of intermediate goods in country \( S \), \( z_{x,S} = 0.05 \); (iii) Scenario 3 (Sc3) – an ad-valorem subsidy to R&D in country \( S \), \( z_{r,S} = 0.05 \); (iv) Scenario 4 (Sc4) – an ad-valorem subsidy to the production of intermediate goods in country \( B \), \( z_{x,B} = 0.05 \).

Table 2 and Figures 1 and 2 show the main results of the simulation made for the mentioned scenarios. Figure 1 shows the path of the technological-knowledge gap, measured by the path of \( \tilde{Q} \): as the gap decreases, \( \tilde{Q} \) increases. Figure 2 shows the effects of the governmental measures on public accounts, measured by the impact on the public deficit to GDP ratio. Finally, Table 2 summarises the initial and final values, in steady-state, for some relevant variables.

The first relevant result to note is that, even without any governmental intervention, the technological-knowledge gap decreases through time (Sc1). This

\[\text{Parameters are chosen to calibrate the steady-state world growth rate around 3% without public intervention. For some parameters the choice is guided by empirical findings, while other parameter values are based on theoretical specifications.}\]
Figure 1: Relative technological-knowledge level of $S$: Sc1 - solid line, Sc2 dashed line, Sc3 dotted line, Sc4 dashdot.
Figure 2: $BuS/Y$ on $S$: Sc2 dashed line, Sc3 dotted line; $BuS/Y$ on $B$: Sc4 dashdot.
<table>
<thead>
<tr>
<th>Scenario</th>
<th>$\tilde{Q}^*$</th>
<th>$r^*$</th>
<th>$g^*$</th>
<th>$B_{B}S_{B}$</th>
<th>$B_{S}S_{B}$</th>
</tr>
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<td>0.0473</td>
<td>0.0308</td>
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<tr>
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<td>0</td>
<td>0.0923</td>
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<tr>
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<td>0.0490</td>
<td>0.0324</td>
<td>0</td>
<td>0.0827</td>
</tr>
<tr>
<td>4</td>
<td>0.7009</td>
<td>0.0491</td>
<td>0.0325</td>
<td>0.0866</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2: Steady-state results

is mainly explained by the level effect resulting from the free trade of intermediate goods, which arises with the creation of the monetary union. This level effect improves the available resources for consumption and investment in the less developed country, affecting then the endogenous force of growth within the union (the R&D imitative and innovative activities).

An ad-valorem subsidy to the production of intermediate goods in the less developed country (Sc2) would clearly improve economic cohesion within the union, by decreasing the technological-knowledge gap ($\tilde{Q}$ increases). This would occur without any relevant negative external effects, as, in line with what is predicted in literature, the interest rate has a very slightly increase. As expected, no major effect occurs on the economic growth rate of the union. Also as expected, this measure leads to the creation of an increasing public deficit, although manageable if such measure would be just temporarily implemented.

An ad-valorem subsidy to R&D in the same country (Sc3) would also improve economic cohesion, but in a lower extent than the previous case: the increase in $\tilde{Q}$ is lower in this case. This direct subsidy to R&D imitative activity has a stronger feedback effect, reflected in a higher growth rate. In this case, the improvement of the state-of-the-art technological-knowledge ($Q_B$) is more powerful: thus, in spite of the increase of $Q_S$, the effect on the economic cohesion is weaker. It is also worth to note that the external effect on the interest rate, although with small importance, is higher than in Sc2. The same kind of impact as in Sc2 is experienced in public accounts.

Sc4 allows us to compare a governmental intervention in $S$ with one in $B$.  

24
The introduction of an ad-valorem subsidy to the production of intermediate goods in $B$ would negatively affect economic cohesion, as it can be seen from the decrease in $\tilde{Q}$ when compared to the case of non-governmental intervention in any country. As expected, external effects would be stronger, when comparing this case to that of Sc1: they include a higher increase in the interest rate which would accompany a higher value for the whole economic growth rate. Also as expected, the situation of public accounts in $B$ would deteriorate, as the public deficit to GDP ratio would continuously increase.

Some relevant conclusions can be raised from the above major results.

(i) The most important effect, in the presence of the possibility of imitation, arises from the removal of all barriers to trade when the monetary union (which necessarily includes a common market) is created. The above mentioned level effect explains the decrease in the technological-knowledge gap in all scenarios, when compared to the case of pre-union and even without any governmental intervention in terms of (direct or indirect) incentive to R&D activities.

(ii) An incentive to imitation/innovation resulting from the governmental intervention in the less developed and small country allows to improve economic cohesion within the union, although with the cost of a disequilibrium in public accounts. Because of weaker feedback effects, the impact of such intervention is higher in the case of a subsidy to the production of intermediate goods than in the case of a direct subsidy to R&D activities. This situation contrasts with the results obtained in a previous work where imitation was not allowed (Afonso and Alves, 2007) and turns to be a major result of the imitation/innovation process.

(iii) The same kind of intervention taken by the most developed and big country would allow a higher growth rate for the whole union. However, such intervention would imply larger costs, namely as economic cohesion would be hurt and external negative effects would be of higher importance.

(iv) More important effects on economic cohesion would be granted if (direct or indirect) incentives to R&D activities would be stronger. However, in such
case the effects on public accounts could be too negative and hard to manage.

(v) The negative effects on public accounts would be attenuated if taxation was allowed. This would mean that the temporal limit for a governmental intervention, such as those described, would probably be enlarged.

(vi) An overall assessment of our results allows for arguing in favour of a temporary differentiation of fiscal rules, in order to allow less developed and small countries to incentive their R&D activities and thus reduce their technological-knowledge gap regarding the most developed countries. Temporary "excessive" public deficits would thus appear as the major cost of an improvement in the economic cohesion of the union.

(vii) The inclusion of incentives to R&D activities as a "pertinent" factor for explaining a worse budgetary behaviour seems to be justified. This turns to be in line with one of the major changes on the framework for fiscal discipline in the eurozone arising from the 2005 reform of the SGP.

4 Concluding remarks

The main purpose of this paper was to analyse if there are sustainable reasons for the existence of different fiscal rules among different countries that form a monetary union and for the consideration of some kind of public expenditure, namely incentives to R&D, as exceptions in what concerns the application of fiscal discipline rules, such as those of the SGP.

To this purpose, we developed a dynamic general-equilibrium growth model with two countries forming a monetary union, one of them being innovator and the other being imitator. Growth is driven by Schumpeterian-R&D applied to intermediate goods which complement labour in each country. In this context, we analysed the effects of a governmental intervention through subsidising (directly or indirectly) R&D activities and compared them to a situation with no governmental intervention. This was done by solving numerically the model and analysing the transitional dynamics towards the steady state.
The results point the removal of all barriers to trade, corresponding to the creation of a common market (a requisite for a monetary union), as the most important factor for a decrease in the technological-knowledge gap between the two countries. That happens because of a level effect when the monetary union is created, which improves resources available for consumption and investment in the less developed country. These resources have feedback effects on the other country, thus affecting the endogenous force of growth within the union, which is R&D innovative activity.

In addition, in the context of a built monetary union, a governmental intervention in order to subsidise (direct or indirectly) R&D activity in the less developed country helps to increase economic cohesion, without having major negative external effects, namely on the interest rate of the union. Contrasting with the previous work by Afonso and Alves (2007), the consideration of imitation and feedback effects leads to the conclusion that a subsidy to the production of intermediate goods works better than a direct subsidy to R&D in order to reduce the technological-knowledge gap. Finally, an intervention conducted by the most developed country hurts economic cohesion and leads to a higher interest rate. In all cases, the economic growth rate of the union increases, although only slightly if the intervention is conducted by the less developed country through a subsidy to the production of intermediate goods.

These results suggest the pertinence of making some exceptions to the European framework for fiscal discipline, namely concerning to expenses leading to more resources devoted to R&D activities. In particular, this would apply to measures taken by the small and less developed countries, in this case also leading to an easier catching-up and to an increase in economic cohesion within the monetary union.
References


